



# One-loop corrections to the Higgs electroweak chiral Lagrangian

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In this talk we study beyond Standard Model scenarios where the Higgs is non-linearly realized. The one-loop ultraviolet divergences of the low-energy effective theory at next-to-leading order,  $\mathcal{O}(p^4)$ , are computed by means of the background-field method and the heat-kernel expansion. The power counting in non-linear theories shows that these divergences are determined by the leading-order effective Lagrangian  $\mathcal{L}_2$ . We focus our attention on the most important  $\mathcal{O}(p^4)$  divergences, which are provided by the loops of Higgs and electroweak Goldstones, as these particles are the only ones that couple through derivatives in  $\mathcal{L}_2$ . The one-loop divergences are renormalized by  $\mathcal{O}(p^4)$  effective operators, and set their running. This implies the presence of chiral logarithms in the amplitudes along with the  $\mathcal{O}(p^4)$  low-energy couplings, which are of a similar importance and should not be neglected in next-to-leading order effective theory calculations, e.g. in composite scenarios.

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## 1. Introduction

In these proceedings we study non-linear electroweak (EW) effective theories including a light Higgs, which we will denote as the electroweak chiral Lagrangian with a light Higgs (ECLh). In Ref. [1] we have computed the next-to-leading order (NLO) corrections induced by scalar boson (Higgs  $h$  and EW Goldstones  $\omega^a$ ) one-loop diagrams. These contributions provide the one-loop corrections to the amplitude that grow with the energy as  $p^4$ , as these particles are the only ones that couple derivatively in the lowest-order (LO) effective Lagrangian [2, 3, 4]. We used the background field method and heat-kernel expansion to extract the ultraviolet divergences of the theory at NLO, i.e.,  $\mathcal{O}(p^4)$ , where  $p$  is the effective field theory (EFT) expansion parameter and refers to any infrared (IR) scale of the EFT –external momenta or masses of the particles in the EFT–. Many beyond Standard Model (BSM) scenarios show this non-linear realization, which are typically strongly coupled theories with composite states [5]. A common feature in non-linear EFT’s is that one-loop corrections are formally of the same order as tree-level contributions from higher dimension operators [6, 7]. Phenomenologically, these two types of corrections are of a similar size, provided the scale of non-linearity  $\Lambda_{\text{non-lin}}$  that suppresses the  $h$  and  $\omega^a$  loops and the composite resonance masses  $M_R$  are similar [8]:

$$\text{NLO tree diagrams} \sim \text{NLO loop diagrams}.$$

## 2. Low-energy Lagrangian and chiral counting

The low-energy theory is given by the usual ingredients [9]:

- **EFT particle content:** the singlet Higgs field  $h$ , the non-linearly realized triplet of EW Goldstones  $\omega^a$  and the SM gauge bosons and fermions.
- **EFT symmetries:** we based our analysis on the symmetry pattern of the SM scalar sector  $\mathcal{G} = SU(2)_L \times SU(2)_R$ , which breaks down spontaneously into the custodial subgroup  $\mathcal{H} = SU(2)_{L+R}$ . The subgroup  $SU(2)_L \times U(1)_Y \in \mathcal{G}$  is gauged.<sup>1</sup>
- **Locality:** The underlying theory may contain non-local exchanges of heavy states. Nevertheless, in the low-energy limit the effective action is provided by an expansion of local operators.

In the case of non-linear Lagrangians the classification of the EFT operators in terms of their canonical dimension is not appropriate and what really ponders the importance of an operator in an observable is their “chiral” scaling  $p^{\hat{d}}$  in terms of the infrared scales  $p$  [3, 7, 10, 11, 12]. The ECLh is organized in the form

$$\mathcal{L}_{\text{ECLh}} = \mathcal{L}_2 + \mathcal{L}_4 + \dots \quad (2.1)$$

where the terms of order  $p^{\hat{d}}$  have the generic form [12, 13]

$$\mathcal{L}_{\hat{d}} \sim \sum_{k, n_F} c_{(\hat{d})} p^{\hat{d}} \left( \frac{\chi}{v} \right)^k \left( \frac{\bar{\psi}\psi}{v^2} \right)^{n_F/2}, \quad (2.2)$$

<sup>1</sup>When fermions are included in the theory  $\mathcal{G}$  must be enlarged to  $\mathcal{G} = SU(2)_L \times SU(2)_R \times U(1)_{\text{B-L}} \supset SU(2)_L \times U(1)_Y$  and  $\mathcal{H} = SU(2)_{L+R} \times U(1)_{\text{B-L}} \supset U(1)_{\text{EM}}$  [10], with B and L the baryon and lepton numbers, respectively.

with  $\hat{d} = d + n_F/2$ ,  $v = (\sqrt{2}G_F)^{-1/2} = 246$  GeV,  $\chi$  ( $\psi$ ) representing any bosonic (fermionic) fields in the ECLh, and being  $p$  any infrared scale appropriately acting on the fields (derivatives, masses of the particles in the EFT, etc.). In the lowest order case  $\hat{d} = 2$  one has couplings  $c^{(2)} \sim v^2$ . The counting can be established more precisely by further classifying what we mean by  $p$  in our operators (explicit derivatives, fermion masses, etc.) [3]. Beyond naive dimensional analysis (NDA), one usually makes further assumptions on the scaling of the coupling of the composite sectors and the elementary fermions. Typically one assumes them to be weakly coupled in order to support the phenomenological observation  $m_\psi \ll 4\pi v \approx 3$  TeV and the moderate size of the Yukawa couplings measured so far at LHC.<sup>2</sup>

The LO Lagrangian is given by [3, 4, 14, 15],

$$\mathcal{L}_2 = \frac{v^2}{4} \mathcal{F}_C \langle u_\mu u^\mu \rangle + \frac{1}{2} (\partial_\mu h)^2 - v^2 V + \mathcal{L}_{YM} + i \bar{\psi} \not{D} \psi - v^2 \langle J_S \rangle, \quad (2.3)$$

where  $\langle \dots \rangle$  stands for the trace of  $2 \times 2$  EW tensors,  $\mathcal{L}_{YM}$  is the Yang–Mills Lagrangian for the gauge fields,  $D_\mu$  is the gauge covariant derivative acting on the fermions,  $V[h/v]$  is the Higgs potential and  $J_S$  denotes the Yukawa operators that couple the SM fermions to  $h$  and  $\omega^a$ . The factors of  $v$  in the normalization of some terms are introduced for later convenience.  $\mathcal{F}_C$ ,  $V$  and  $J_S$  are functionals of  $x = h/v$ , and have Taylor expansions,  $\mathcal{F}_C[x] = 1 + 2ax + bx^2 + \dots$ ,  $J_S[x] = \sum_n J_S^{(n)} x^n / n!$  and  $V[x] = m_h^2 \left( \frac{1}{2} x^2 + \frac{1}{2} d_3 x^3 + \frac{1}{8} d_4 x^4 + \dots \right)$ , given in terms of the constants  $a, b, m_h$ , etc. [3, 4, 14, 15]. In the non-linear realization of the spontaneous EW symmetry breaking, the Goldstones are parameterized through the coordinates  $(u_L, u_R)$  of the  $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$  coset space [20], with the  $SU(2)$  matrices  $u_{L,R}$  being functions of the Goldstone fields  $\omega^a$  which enter through the building blocks

$$\begin{aligned} u_\mu &= iu_R^\dagger (\partial_\mu - ir_\mu) u_R - iu_L^\dagger (\partial_\mu - i\ell_\mu) u_L, \quad \Gamma_\mu = \frac{1}{2} u_R^\dagger (\partial_\mu - ir_\mu) u_R + \frac{1}{2} u_L^\dagger (\partial_\mu - i\ell_\mu) u_L, \\ \nabla_\mu \cdot &= \partial_\mu \cdot + [\Gamma_\mu, \cdot], \quad f_\pm^{\mu\nu} = u_L^\dagger \ell^{\mu\nu} u_L \pm u_R^\dagger r^{\mu\nu} u_R, \quad r_{\mu\nu} = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \quad (R \leftrightarrow L), \\ J_S &= J_{YRL} + J_{YRL}^\dagger, \quad J_P = i(J_{YRL} - J_{YRL}^\dagger), \quad J_{YRL} = -\frac{1}{\sqrt{2}v} u_R^\dagger \hat{Y} \psi_R^\alpha \bar{\psi}_L^\alpha u_L, \end{aligned} \quad (2.4)$$

with  $\psi_{R,L} = \frac{1}{2}(1 \pm \gamma^5)\psi$  and the  $SU(2)$  doublet  $\psi = (t, b)^T$ . The summation over the Dirac index  $\alpha$  in  $\psi_{Rm}^\alpha \bar{\psi}_{Ln}^\alpha = -\bar{\psi}_{Ln}^\alpha \psi_{Rm}^\alpha$  is assumed and its tensor structure under  $\mathcal{G}$  and indices  $m$  and  $n$  are left implicit. The  $2 \times 2$  matrix  $\hat{Y}[h/v]$  is a spurion auxiliary field, functional of  $h/v$ , which incorporates the fermionic Yukawa coupling [3, 10, 18]. Other SM fermion doublets and the flavour symmetry breaking between generations can be incorporated by adding in  $J_{YRL}$  an additional family index in the fermion fields,  $\psi^A$ , and promoting  $\hat{Y}$  to a tensor  $\hat{Y}^{AB}$  in the generation space [18]. In our analysis,  $\ell_\mu, r_\mu, \hat{Y}$  are spurion auxiliary background fields that keep the invariance of the ECLh action under  $\mathcal{G}$ . When evaluating physical matrix elements, custodial symmetry is then explicitly broken in the same way as in the SM, keeping only the gauge invariance under the subgroup  $SU(2)_L \times U(1)_Y \subset \mathcal{G}$  [2, 3, 4, 14], with the auxiliary fields taking the value  $\ell_\mu = -\frac{g}{2} W_\mu^a \sigma^a$ ,  $r_\mu = -\frac{g'}{2} B_\mu \sigma^3$ ,  $\hat{Y}[h/v] = \hat{y}_t[h/v] P_+ + \hat{y}_b[h/v] P_-$ , with  $P_\pm = (1 \pm \sigma^3)/2$ .

<sup>2</sup>Based on pure NDA the Yukawa operators in the ECLh would be  $\mathcal{O}(p^1)$ , spoiling the chiral power expansion. In order to avoid this, one needs to make the phenomenologically supported assumption that the constants  $\lambda_\psi$  that parametrize the couplings between the elementary fermions and the composite scalars  $h$  and  $\omega^a$  are further suppressed, scaling at least like  $\lambda_\psi \sim \mathcal{O}(p)$  or higher in the chiral counting [3, 13].

In order to compute the one-loop fluctuations we will consider the coset representatives  $u_L = u_R^\dagger = u$ , often expressed in the exponential parametrization  $U = u^2 = \exp\{i\omega^a \sigma^a / v\}$ .<sup>3</sup>

The IR scales in the low-energy theory are

$$\partial_\mu, \quad r_\mu, \quad \ell_\mu, \quad m, \quad g^{(\prime)} v, \quad \hat{Y} v \quad \sim \quad \mathcal{O}(p), \quad (2.5)$$

with  $m = m_{h,W,Z,\psi}$ . Accordingly, covariant derivatives scale as the ordinary ones [6] and the Lagrangian is invariant under  $\mathcal{G}$  at every order in the chiral expansion. Based on this we are going to sort out the building blocks and operators according to the assignment [7, 11],

$$\begin{aligned} \frac{\chi}{v} &\sim \mathcal{O}(p^0) && \text{(for the boson fields } \chi = h, \omega^a, W_\mu^a, B_\mu), \\ \frac{\psi}{v} &\sim \mathcal{O}(p^{1/2}) && \text{(for the fermion fields } \psi = t, b, \text{ etc}). \end{aligned} \quad (2.6)$$

Therefore, the chiral order of the various building blocks reads

$$\mathcal{F}_C \sim \mathcal{O}(p^0), \quad u_\mu, \nabla_\mu \sim \mathcal{O}(p^1), \quad r_{\mu\nu}, \ell_{\mu\nu}, f_{\pm}^{\mu\nu}, J_{YRL}, J_S, J_P, V \sim \mathcal{O}(p^2). \quad (2.7)$$

Hence, the Lagrangian in Eq. (2.3) is  $\mathcal{O}(p^2)$  and provides the LO.

So far all we did was to sort out the possible terms of the EFT Lagrangian assigning them an order. The relevance of this classification is that, at low energies, when one computes the contributions to a given process the more important ones are given by the Lagrangian operators with a lower chiral dimension. An arbitrary diagram with vertices from  $\mathcal{L}_{\hat{d}}$  behaves at low energies like [3, 7, 11, 12, 13]

$$\mathcal{M} \sim \frac{p^2}{v^{E-2}} \left( \frac{p^2}{16\pi^2 v^2} \right)^L \prod_{\hat{d}} \left( \frac{c_{(\hat{d})} p^{\hat{d}-2}}{v^2} \right)^{N_{\hat{d}}}, \quad (2.8)$$

with the IR scales  $p$ ,  $L$  the number of loops,  $N_{\hat{d}}$  the number of subleading vertices from  $\mathcal{L}_{\hat{d}>2}$  (with coupling  $c_{(\hat{d})}$ ) and  $E$  the number of external legs. We can have an arbitrary number of  $\mathcal{L}_2$  vertices in the diagram and the amplitude will still have the same scaling with  $p$ , provided the number of loops is fixed. If we add vertices of a higher chiral dimension we will increase the scaling of the diagram with  $p$ . Thus, we have that the one-loop corrections with only  $\mathcal{L}_2$  vertices are  $\mathcal{O}(p^4)$  and their UV divergences are cancelled out by tree-level diagrams that contain one  $\mathcal{L}_4$  vertex: the renormalization of the effective couplings at  $\mathcal{O}(p^4)$  will render the effective action finite at NLO.

### 3. Fluctuations around a background field

We are going to consider perturbations  $\eta$  in the fields around their equations of motion (EoM) solutions. The Lagrangian in the integrand of the generating functional has also a corresponding expansion in the perturbation, where each order in  $\eta$  contains relevant information [19]:

$$\mathcal{L} = \underbrace{\mathcal{L}^{\mathcal{O}(\eta^0)}}_{\text{Tree-level}} + \underbrace{\mathcal{L}^{\mathcal{O}(\eta^1)}}_{\text{EoM}} + \underbrace{\mathcal{L}^{\mathcal{O}(\eta^2)}}_{\text{1-loop}} + \underbrace{\mathcal{O}(\eta^3)}_{\text{Higher loops}}. \quad (3.1)$$

<sup>3</sup>Other Goldstone parametrizations are discussed in [6, 12, 16, 17], being all of them fully equivalent when describing on-shell matrix elements.

The Lagrangian evaluated at the classical solution provides the tree-level contributions to the effective action, the requirement that the linear term in the  $\eta$  expansion vanishes provides the EoM, the quadratic fluctuation in  $\eta$  provides the 1-loop contributions to the effective action and higher loops are encoded in the remaining terms of the  $\eta$  expansion.

In our analysis [1] we were interested in the one-loop UV divergences at  $\mathcal{O}(p^4)$ , that is those coming from diagrams with  $\mathcal{L}_2$  vertices. We studied the structures that grow faster with the energy at one-loop, as  $(\text{Energy})^4$ . They were given by the loops of scalars ( $h$  and  $\omega^a$ ) which are the only ones that couple derivatively in  $\mathcal{L}_2$ . We made the Goldstone representative choice  $u_L = u_R^\dagger$  [21] and performed fluctuations of the scalar fields (Higgs and Goldstones) around the classical background fields  $\bar{h}$  and  $\bar{u}_{L,R}$  [1]:

$$u_{R,L} = \bar{u}_{R,L} \exp \left\{ \pm i \mathcal{F}_C^{-1/2} \Delta / (2v) \right\}, \quad h = \bar{h} + \varepsilon, \quad (3.2)$$

with  $\Delta = \Delta^a \sigma^a$ . Without any loss of generality we introduced the factor  $\mathcal{F}_C^{-1/2}$  in the exponent for later convenience, allowing us to write down the second-order fluctuation of the action in the canonical form [19]. To obtain the one-loop effective action within the background field method we then retained the quantum fluctuations  $\bar{\eta}^T = (\Delta^a, \varepsilon)$  up to quadratic order [19].

Since we are interested in the loops with only  $\mathcal{L}_2$  vertices, we study the  $\eta$  expansion of the LO Lagrangian  $\mathcal{L}_2^{\mathcal{O}(\eta^0)} = \mathcal{L}_2[\bar{u}_{L,R}, \bar{h}]$ . The tree-level effective action is equal to the action evaluated at the classical solution,  $\int d^D x \mathcal{L}^{\mathcal{O}(\eta^0)}$ .

### 3.1 $\mathcal{O}(\eta^1)$ fluctuations: EoM

The background field configurations correspond to the solutions of the classical equations of motion (EoM), defined by the requirement that the linear term,

$$\mathcal{L}_2^{\mathcal{O}(\eta^1)} = \frac{v}{2} \langle \Delta (\nabla^\mu (\mathcal{F}_C u_\mu) + 2 \mathcal{F}_C J_P) \rangle + v \varepsilon \left( \frac{1}{4} \mathcal{F}_C' \langle u_\mu u^\mu \rangle - \frac{\partial^2 h}{v} - V' - \langle J_S' \rangle \right), \quad (3.3)$$

vanishes for arbitrary  $\bar{\eta}^T = (\Delta^a, \varepsilon)$ . This yields the EoM,

$$\nabla^\mu u_\mu = -2 \mathcal{F}_C^{-1} J_P - u_\mu \partial^\mu (\ln \mathcal{F}_C), \quad \frac{\partial^2 h}{v} = \frac{1}{4} \mathcal{F}_C' \langle u_\mu u^\mu \rangle - V' - \langle J_S' \rangle. \quad (3.4)$$

Here and in the following, we abuse of the notation by writing the background fields  $\bar{u}_{L,R}$  and  $\bar{h}$  as  $u_{L,R}$  and  $h$  for conciseness.

### 3.2 $\mathcal{O}(\eta^2)$ fluctuations: 1-loop corrections

The  $\mathcal{O}(\eta^2)$  term of the expansion of  $\mathcal{L}_2$  reads

$$\begin{aligned} \mathcal{L}_2^{\mathcal{O}(\Delta^2)} &= -\frac{1}{4} \langle \Delta \nabla^2 \Delta \rangle + \frac{1}{16} \langle [u_\mu, \Delta] [u^\mu, \Delta] \rangle \\ &+ \left[ \frac{\mathcal{F}_C^{-\frac{1}{2}} \mathcal{K}}{8} \left( \frac{\partial^2 h}{v} \right) + \frac{\Omega}{16} \left( \frac{\partial_\mu h}{v} \right)^2 \right] \langle \Delta^2 \rangle + \frac{1}{2 \mathcal{F}_C} \langle \Delta^2 J_S \rangle, \\ \mathcal{L}_2^{\mathcal{O}(\varepsilon^2)} &= -\frac{1}{2} \varepsilon \left[ \partial^2 - \frac{1}{4} \mathcal{F}_C'' \langle u_\mu u^\mu \rangle + V'' + \langle J_S'' \rangle \right] \varepsilon, \\ \mathcal{L}_2^{\mathcal{O}(\varepsilon \Delta)} &= -\frac{1}{2} \varepsilon \mathcal{F}_C' \langle u_\mu \nabla^\mu (\mathcal{F}_C^{-\frac{1}{2}} \Delta) \rangle + \mathcal{F}_C^{-\frac{1}{2}} \varepsilon \langle \Delta J_P' \rangle, \end{aligned} \quad (3.5)$$

in terms of  $\mathcal{K} = \mathcal{F}_C^{-1/2} \mathcal{F}'_C$  and  $\Omega = 2\mathcal{F}_C''/\mathcal{F}_C - (\mathcal{F}'_C/\mathcal{F}_C)^2$ . Through a proper definition of the differential operator  $d_\mu \vec{\eta} = \partial_\mu \vec{\eta} + Y_\mu \vec{\eta}$ , one can rewrite  $\mathcal{L}_2^{\mathcal{O}(\eta^2)}$  in the canonical form

$$\mathcal{L}_2^{\mathcal{O}(\eta^2)} = -\frac{1}{2} \vec{\eta}^T (d_\mu d^\mu + \Lambda) \vec{\eta}, \quad (3.6)$$

where  $d_\mu$  and  $\Lambda$  depend on  $h$ ,  $u_{L,R}$  and on the gauge boson and fermion fields (see App. A in Ref. [1]). They scale according to the chiral counting as

$$d_\mu \sim \mathcal{O}(p), \quad \Lambda \sim \mathcal{O}(p^2). \quad (3.7)$$

The quadratic form (3.6) yields a Gaussian integration over  $\vec{\eta}$  in the path-integral, which gives the one-loop contribution to the effective action,

$$S^{1\ell} = \frac{i}{2} \text{tr} \log (d_\mu d^\mu + \Lambda). \quad (3.8)$$

where tr stands for the full trace of the operator, including the trace in the adjoint representation of the flavour space and that in the coordinate space.

The computation of the full one-loop effective action is in general a difficult task. However, it is easier to extract its UV-divergent part.<sup>4</sup> For this, we first transform the trace of the log in configuration space into an integral of an exponential by means of the Schwinger-DeWitt proper-time representation embedded in the heat-kernel expansion [19]:

$$\begin{aligned} \langle x | \log (d_\mu d^\mu + \Lambda) | x \rangle &= - \int_0^\infty \frac{d\tau}{\tau} \underbrace{\langle x | e^{-\tau(d_\mu d^\mu + \Lambda)} | x \rangle}_{\equiv H(x, \tau)} + C \\ &\stackrel{\text{dim-reg}}{=} - \frac{i}{(4\pi)^{D/2}} \sum_{n=0}^\infty m^{D-2n} \Gamma\left(n - \frac{D}{2}\right) a_n(x) + C, \end{aligned} \quad (3.9)$$

where we obtain the expansion in term of local operators of increasing dimension given by the Seeley-DeWitt coefficients  $a_n(x)$  and the potential UV divergences are contained in the Gamma function for  $D \rightarrow 4$ . Only the terms of the series with  $2n \leq D$  are divergent and they have their origin on the short-distance part of the integral, this is, in its lower limit  $\tau \rightarrow 0$  (the integration variable  $\tau$  has dimensions of length-square in natural units). The (infinite) constant  $C$  is irrelevant here. In the second line we have used the Fourier decomposition of the heat-kernel in momentum space,

$$H(x, \tau) = \langle x | e^{-\tau(d_\mu d^\mu + \Lambda)} | x \rangle = \int \frac{d^D p}{(2\pi)^D} e^{-ipx} e^{-\tau(d_\mu d^\mu + \Lambda)} e^{ipx} = \frac{ie^{-\tau m^2}}{(4\pi\tau)^{D/2}} \sum_{n=0}^\infty a_n(x) \tau^n, \quad (3.10)$$

where the coefficients  $a_n(x)$  are extracted by expanding the interaction part of  $(d_\mu d^\mu + \Lambda)$  in the exponential in powers of  $\tau$  and performing the integral of each corresponding term in dimensional regularization. In our case, the residue of the  $(D-4)^{-1}$  pole is given by the trace  $\text{Tr}\{a_2(x)\}$  [1,

<sup>4</sup>It is also sometimes possible to compute the effective potential. See for instance [25].

19]:<sup>5</sup>

$$\begin{aligned}
S^{1\ell} &= -\frac{\mu^{D-4}}{16\pi^2(D-4)} \int d^D x \text{Tr}\{a_2(x)\} + \text{finite} \\
&= -\frac{\mu^{D-4}}{16\pi^2(D-4)} \int d^D x \text{Tr} \left\{ \frac{1}{12} [d_\mu, d_\nu] [d^\mu, d^\nu] + \frac{1}{2} \Lambda^2 \right\} + \text{finite} \\
&= -\frac{\mu^{D-4}}{16\pi^2(D-4)} \int d^D x \sum_k \Gamma_k \mathcal{O}_k + \text{finite}, \tag{3.11}
\end{aligned}$$

where Tr refers to the trace over the  $4 \times 4$  operators that acted on the fluctuation vector  $\vec{\eta}$  in Eq. (3.6). The UV-divergence is determined by the non-derivative quadratic fluctuation  $\Lambda$  and the differential operator  $d_\mu$  through  $[d_\mu, d_\nu] = Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu + [Y_\mu, Y_\nu]$ , with both  $\Lambda, Y_{\mu\nu} \sim \mathcal{O}(p^2)$ . By looking at the second line of (3.11) it is then clear that the UV-divergences that appear from one-loop diagrams with  $\mathcal{L}_2$  vertices are  $\mathcal{O}(p^4)$  and that they require counterterms of that order to be cancelled out. However, as some of this  $p$  factors are actually constants (e.g., Higgs masses  $m_h$ ) the structure of the operators  $\mathcal{O}_k$  resembles that of other operators already present in  $\mathcal{L}_2$ . In general, the one-loop UV-divergences in the effective action will have a local structure and can be written in terms of the basis of operators of chiral dimension  $p^2, p^4$ , etc. [2, 14, 3, 4],

$$S^{1\ell, \infty} = \int d^D x \left( \mathcal{L}_2^{1\ell, \infty} + \mathcal{L}_4^{1\ell, \infty} + \dots \right). \tag{3.12}$$

Let us summarize the results for the NLO UV-divergences from  $h$  and  $\omega^a$  loops:

- **UV divergences with the structure of the  $\mathcal{L}_2$  operators in Eq. (2.3):**

$$\begin{aligned}
\mathcal{L}_2^{1\ell, \infty} &= -\frac{\mu^{D-4}}{16\pi^2(D-4)} \left\{ \frac{1}{8} \left[ \frac{\mathcal{F}'_C V'}{\mathcal{F}_C} (4 - \mathcal{K}^2) - \mathcal{F}_C \Omega V'' \right] \langle u_\mu u^\mu \rangle \right. \\
&\quad - \frac{3\mathcal{F}'_C V' \Omega}{8\mathcal{F}_C} \left( \frac{\partial_\mu h}{v} \right)^2 + \left[ \frac{1}{2} (V'')^2 + \frac{3\mathcal{K}^2}{8\mathcal{F}_C} (V')^2 \right] \\
&\quad \left. + \left( V'' \langle J'_S \rangle - \frac{3\mathcal{F}'_C V'}{2\mathcal{F}_C} \langle \Gamma_S \rangle \right) \right\}, \tag{3.13}
\end{aligned}$$

where  $\Gamma_S = \mathcal{F}_C^{-1} (J_S - \mathcal{F}'_C J'_S / 2)$  is an  $\mathcal{O}(p^2)$  tensor. These UV divergences are cancelled out through the renormalization of various parts of  $\mathcal{L}_2$ : the couplings in the  $\mathcal{F}_C$  term (1st line); the Higgs kinetic term (1st term in 2nd line), which requires a NLO Higgs field redefinition; the coefficients of the Higgs potential, e.g. the Higgs mass (2nd bracket in 2nd line); the Yukawa term couplings in  $\hat{Y}$  (3rd line).

- **UV divergences with the structure of the  $\mathcal{L}_4$  operators:** the  $\mathcal{L}_4^{1\ell, \infty}$  terms are further classified here into two types, according to whether they include fermion fields or not.

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<sup>5</sup>The IR regulator  $m$  of the heat-kernel integral can be made arbitrary small and hence the term  $\text{Tr}\{a_1(x)\} = -\text{Tr}\{\Lambda\}$  does not contribute to the UV divergent part; note that the particle masses are accounted as a perturbation, i.e., within  $\Lambda(x)$ .



$c_k$	Operator $\mathcal{O}_k$	$\Gamma_k$	$\Gamma_k[0]$
$c_1$	$\frac{1}{4}\langle f_+^{\mu\nu} f_{+\mu\nu} - f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$\frac{1}{24}(\mathcal{K}^2 - 4)$	$-\frac{1}{6}(1 - a^2)$
$(c_2 - c_3)$	$\frac{i}{2}\langle f_+^{\mu\nu} [u_\mu, u_\nu] \rangle$	$\frac{1}{24}(\mathcal{K}^2 - 4)$	$-\frac{1}{6}(1 - a^2)$
$c_4$	$\langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$	$\frac{1}{96}(\mathcal{K}^2 - 4)^2$	$\frac{1}{6}(1 - a^2)^2$
$c_5$	$\langle u_\mu u^\mu \rangle^2$	$\frac{1}{192}(\mathcal{K}^2 - 4)^2 + \frac{1}{128}\mathcal{F}_C^2\Omega^2$	$\frac{1}{8}(a^2 - b)^2 + \frac{1}{12}(1 - a^2)^2$
$c_6$	$\frac{1}{v^2}(\partial_\mu h)(\partial^\mu h) \langle u_\nu u^\nu \rangle$	$\frac{1}{16}\Omega(\mathcal{K}^2 - 4) - \frac{1}{96}\mathcal{F}_C\Omega^2$	$-\frac{1}{6}(a^2 - b)(7a^2 - b - 6)$
$c_7$	$\frac{1}{v^2}(\partial_\mu h)(\partial_\nu h) \langle u^\mu u^\nu \rangle$	$\frac{1}{24}\mathcal{F}_C\Omega^2$	$\frac{2}{3}(a^2 - b)^2$
$c_8$	$\frac{1}{v^4}(\partial_\mu h)(\partial^\mu h)(\partial_\nu h)(\partial^\nu h)$	$\frac{3}{32}\Omega^2$	$\frac{3}{2}(a^2 - b)^2$
$c_9$	$\frac{(\partial_\mu h)}{v} \langle f_-^{\mu\nu} u_\nu \rangle$	$\frac{1}{24}\mathcal{F}'_C\Omega$	$-\frac{1}{3}a(a^2 - b)$
$c_{10}$	$\frac{1}{2}\langle f_+^{\mu\nu} f_{+\mu\nu} + f_-^{\mu\nu} f_{-\mu\nu} \rangle$	$-\frac{1}{48}(\mathcal{K}^2 + 4)$	$-\frac{1}{12}(1 + a^2)$

**Table 1:** Purely bosonic operators needed for the renormalization of the NLO effective Lagrangian  $\mathcal{L}_4$  [1]. In the last column, we provide the first term  $\Gamma_k[0]$  in the expansion of the  $\Gamma_k$  in powers of  $(h/v)$  by using  $\mathcal{F}_C = 1 + 2ah/v + bh^2/v^2 + \mathcal{O}(h^3)$ . The first five operators  $\mathcal{O}_i$  have the structure of the respective  $a_i$  Longhitano operator [2, 23] (with  $i = 1 \dots 5$ ). In addition,  $c_6 = \mathcal{F}_{D7}$ ,  $c_7 = \mathcal{F}_{D8}$  and  $c_8 = \mathcal{F}_{D11}$  in the notation of Ref. [3]. The last operator of the list,  $\mathcal{O}_{10} = 2\langle r_{\mu\nu}r^{\mu\nu} + \ell_{\mu\nu}\ell^{\mu\nu} \rangle$ , only depends on the EW field strength tensors and its coefficient is labeled as  $c_{10} = H_1$  in the notation of Ref. [6]. In the notation of [13]  $c_{10} = \mathcal{F}_2$ ,  $c_2 - c_3 = \mathcal{F}_3$  and  $c_k = \mathcal{F}_k$  for  $k \neq 2, 3$ .

### 1. Fermionic operators $\mathcal{L}_4^{1\ell, \infty}|_{\text{Fer}}$ :

$$\begin{aligned}
\mathcal{L}_4^{1\ell, \infty}|_{\text{Fer}} = & -\frac{\mu^{D-4}}{16\pi^2(D-4)} \left\{ \left\langle \left( \frac{\mathcal{K}^2}{4} - 1 \right) \Gamma_S - \frac{\mathcal{F}_C\Omega}{8} J_S'' \right\rangle \langle u^\mu u_\mu \rangle \right. \\
& + \frac{3}{4}\Omega \langle \Gamma_S \rangle \left( \frac{\partial_\mu h}{v} \right)^2 + \frac{1}{2}\Omega \langle \Gamma_P u^\mu \rangle \left( \frac{\partial_\mu h}{v} \right) \\
& \left. + \frac{1}{2}\langle J_S'' \rangle^2 + \frac{3}{2}\langle \Gamma_S \rangle^2 + \frac{1}{\mathcal{F}_C} (2\langle \Gamma_P^2 \rangle - \langle \Gamma_P \rangle^2) \right\}, \tag{3.14}
\end{aligned}$$

with  $\Gamma_P = J'_P - \mathcal{F}_C^{-1}\mathcal{F}'_C J_P/2$  being an  $\mathcal{O}(p^2)$  tensor.

2. **Purely bosonic  $\mathcal{O}(p^4)$  divergences  $\mathcal{L}_4^{1\ell, \infty}|_{\text{Bos}}$ :** This is actually the main result of our computation as these  $\mathcal{O}(p^4)$  operators of the effective action can be only produced from the derivative interactions in  $\mathcal{L}_2$ . They spoil the renormalizability of the SM Lagrangian and lead to the appearance of “true”  $\mathcal{O}(p^4)$  UV-divergences, this is, operators with 4 covariant derivatives.<sup>6</sup> The outcome is summarized in Table 1.

<sup>6</sup>In the case that the covariant derivatives act on  $h$  they just reduce to partial derivatives. Note that the field-strength tensors can be always realized as the commutator of two covariant derivatives.



One can observe that the non-linearity of the  $\mathcal{L}_2$  Lagrangian (where, in general,  $h$  is not introduced via a complex double  $\Phi$ ) is the origin of these higher-dimension divergences. For  $\mathcal{F}_C[h/v] = 1 + 2ah/v + bh^2/v^2 + \dots$ , the combinations  $\mathcal{K}$  and  $\Omega$  that rule the structure of the divergences are given by  $(\mathcal{K}^2 - 4) = 4(a - 1) + \mathcal{O}(h/v)$ ,  $\Omega = 4(b - a^2) + \mathcal{O}(h/v)$ . In particular in the linear limit  $\mathcal{F}_C = (1 + h/v)^2$  and  $\hat{Y}[h/v] = Y(1 + h/v)$ , all the  $\mathcal{O}(p^4)$  divergences from  $h$  and  $\omega^a$  loops disappear,

$$(\mathcal{K}^2 - 4) = \Omega = 0, \quad J_S'' = \Gamma_S = \Gamma_P = 0, \quad (3.15)$$

where the cancellation in the first identity relies only on the form of  $\mathcal{F}_C$ , and the second one – related to four-fermion operators in  $\mathcal{L}_4^{1\ell,\infty}$  – requires also the linear structure in  $\hat{Y}$ .

#### 4. Renormalization at NLO in the ECLh

In order to have a finite 1-loop effective action the divergences in Eq. (3.11) are canceled by the counterterms

$$\mathcal{L}^{\text{ct}} = \sum_k c_k \mathcal{O}_k, \quad (4.1)$$

such that  $\mathcal{L}^{\text{ct}} + \mathcal{L}^{1\ell,\infty} = \text{finite}$ , where the  $\mathcal{O}_k$  is the previous basis of EFT operators, translating into the renormalization conditions

$$c_k = c_k^r + \frac{\mu^{D-4}}{16\pi^2(D-4)} \Gamma_k. \quad (4.2)$$

This leads to the renormalization group equations (RGE) for the  $\mathcal{O}(p^4)$  coupling constants,

$$\frac{dc_{k,n}^r}{d \ln \mu} = -\frac{\Gamma_{k,n}}{16\pi^2}, \quad \text{with} \quad \Gamma_k[h/v] = \sum_n \frac{\Gamma_{k,n}}{n!} \left(\frac{h}{v}\right)^n, \quad c_k[h/v] = \sum_n \frac{c_{k,n}}{n!} \left(\frac{h}{v}\right)^n. \quad (4.3)$$

Physically, this means that the NLO effective couplings will appear in the amplitudes in combinations with logarithms of IR scales  $p$ .

#### 5. Conclusions

Modifying the LO Lagrangian of our EFT action by allowing a non-linear structure for the Higgs field ( $\mathcal{F}_C \neq (1 + h/v)^2$ ) has important implications not only at lowest order but also at NLO. Any misalignment between Higgs  $h$  and Goldstone fields  $\omega^a$  that does not allow us to combine them into a complex doublet  $\Phi$  produces a whole new set of divergences absent in linear theory. Nevertheless, it is possible to find combinations of couplings that are renormalization group invariant (RGI). Some examples derived in [1] are the couplings that determine  $\gamma\gamma \rightarrow ZZ, Z\gamma, \gamma\gamma, \gamma\gamma, Z\gamma \rightarrow h, hh, hhh\dots$  Some of the latter RGI relations were known from previous works ( $\gamma\gamma \rightarrow ZZ$  [12],  $\gamma\gamma, Z\gamma \rightarrow h$  [12, 24]). In addition our result also reproduces the running found in  $WW, ZZ$  and  $hh$  scattering [16, 26].

Our computation in [1] did not address the following two issues: on the one hand, the analysis of the additional contributions to the Higgs potential and their phenomenological implications; on the other hand, deviations from the linear-Higgs scenario leads to the appearance of

UV-divergences and logs related to four-fermion operators, e.g.  $\langle J_{S,P}^2 \rangle$ , which could be strongly constrained by flavour tests. The study of the latter, together with the full NLO computation including gauge boson and fermion loops, is postponed for future work.

## References

- [1] F.-K. Guo, P. Ruiz-Femenía and J.J. Sanz-Cillero, Phys.Rev. D **92** (2015) 7, 074005.
- [2] A.C. Longhitano, Phys. Rev. D **22** (1980) 1166; Nucl. Phys. B **188** (1981) 118; T. Appelquist and C. Bernard, Phys.Rev. D **22** (1980) 200.
- [3] G. Buchalla and O. Catà, JHEP **1207** (2012) 101; G. Buchalla, O. Catà and C. Krause, Phys.Lett. B **731** (2014) 80; Nucl.Phys. B **880** (2014) 552.
- [4] R. Alonso *et al.*, Phys.Lett. B **722** (2013) 330; 726 (2013) 926.
- [5] R. Contino, [arXiv:1005.4269 [hep-ph]].
- [6] J. Gasser and H. Leutwyler, Annals Phys. **158** (1984) 142; Nucl. Phys. B **250** (1985) 465.
- [7] Steven Weinberg, Physica A **96** (1979) 327.
- [8] J.J. Sanz-Cillero, [arXiv:1509.09116 [hep-ph]].
- [9] A. V. Manohar, Lect.Notes Phys. **479** (1997) 311; A. Pich, [arXiv:hep-ph/9806303].
- [10] J. Hirn and J. Stern, Eur.Phys.J. C **34** (2004) 447; JHEP **0409** (2004) 058.
- [11] H. Georgi and A. Manohar Nucl.Phys. B **234** (1984) 189.
- [12] R.L. Delgado, A. Dobado, M.J. Herrero and J.J. Sanz-Cillero, JHEP **1407** (2014) 149.
- [13] A. Pich *et al.*, [arXiv:1510.03114 [hep-ph]]; A. Pich, I. Rosell, J. Santos and J.J. Sanz-Cillero, in preparation.
- [14] B. Grinstein and M. Trott, Phys.Rev. D **76** (2007) 073002.
- [15] G.F. Giudice *et al.*, JHEP **0706** (2007) 045.
- [16] R.L. Delgado, A. Dobado, F.J. Llanes-Estrada, JHEP **1402** (2014) 121.
- [17] M.B. Gavela *et al.*, JHEP **1503** (2015) 043.
- [18] G. D’Ambrosio *et al.*, Nucl.Phys. B **645** (2002) 155.
- [19] G. ‘t Hooft, Nucl.Phys. B **62** (1973) 444; P. Ramond, Front.Phys. **74** (1989) 1; B.S. DeWitt, Int.Ser.Monogr.Phys. **114** (2003) 1; A.O. Barvinsky and G.A. Vilkovisky, Phys.Rept. **119** (1985) 1; C.Lee, T. Lee and H. Min, Phys.Rev. D **39** (1989) 1681; R.D. Ball, Phys.Rept. **182** (1989) 1; D.V. Vassilevich, Phys. Rept. **388** (2003) 279.
- [20] C.G. Callan, Jr. *et al.*, Phys.Rev. **177** (1969) 2247; S.R. Coleman, J. Wess and B. Zumino, Phys.Rev. **177** (1969) 2239.
- [21] A. Pich, I. Rosell and J.J. Sanz-Cillero, Phys.Rev.Lett. **110** (2013) 181801; JHEP **1401** (2014) 157.
- [22] G. Buchalla *et al.*, Phys.Lett. B **750** (2015) 298.
- [23] M.J. Herrero and E. Ruiz-Morales, Nucl.Phys. B **418** (1994) 431; **437** (1995) 319.
- [24] A. Azatov *et al.*, Phys.Rev. D **88** (2013) 7, 075019.
- [25] F. K. Guo and U.-G. Meißner, Phys. Lett. B **749** (2015) 278.
- [26] D. Espriu, F. Mescia and B. Yenko, Phys.Rev. D **88** (2013) 055002.